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RESEARCH IN CALCULUS OF VARIATIONS NUMERICAL ANALYSIS AND RELATED FIELDS

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Final Report

M. R. Hestenes

August 31, 1975

U. S. ARMY RESEARCH OFFICE

Contract DA-31-124-ARO-D-355 Grant DA-ARO-D-31-124-71-G18

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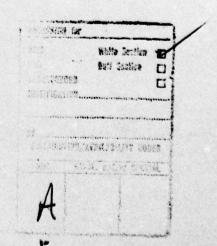
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